1) Which of the following integrals are improper? Why?

a) $\int_1^2 \frac{1}{2x-1}\,dx$ No, continuous.

b) $\int_0^1 \frac{1}{2x-1}\,dx$ Yes, infinite discontinuity.

c) $\int_{-\infty}^{\infty} \frac{\sin x}{1+x^2}\,dx$ Yes, infinite interval.

d) $\int_1^2 \ln(x-1)\,dx$ Yes, infinite discontinuity.

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

2) $\int_1^{\infty} \frac{1}{(3x+1)^2}\,dx$ Convergent, $\frac{1}{12}$

3) $\int_{-\infty}^{0} \frac{1}{2x-5}\,dx$ Divergent, $-\infty$

4) $\int_0^{\infty} \frac{x}{(x^2+2)^2}\,dx$ $\boxed{\text{Convergent, } \frac{1}{4}}$

5) $\int_{-\infty}^{-1} e^{-2t}\,dt$ $\boxed{\text{Divergent, } \infty}$

6) $\int_{-\infty}^{\infty} \frac{x\,dx}{1+x^2}$ $\boxed{\text{Divergent, } -\infty}$

7) $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ $\boxed{\text{Convergent, } 0}$

8) $\int_{-\infty}^{\infty} e^{-|x|}\,dx$ $\boxed{\text{Convergent, } 2}$

9) $\int_{0}^{\infty} se^{-5s}\,ds$ $\boxed{\text{Convergent, } \frac{1}{25}}$

10) $\int_{-\infty}^{6} re^{r/3}\, dr$ $\boxed{\text{Convergent, } 9e^2}$

11) $\int_{1}^{\infty} \frac{\ln x}{x^3}\, dx$ $\boxed{\text{Convergent, } 0}$

12) $\int_{0}^{3} \frac{dx}{\sqrt{x}}$ $\boxed{\text{Convergent, } 2\sqrt{3}}$

13) $\int_{-1}^{0} \frac{dx}{x^2}$ $\boxed{\text{Divergent, } \infty}$

14) $\int_{1}^{9} \frac{dx}{\sqrt[3]{x-9}}$ $\boxed{\text{Convergent, } -6}$

15) $\int_{-2}^{3} \frac{dx}{x^4}$ $\boxed{\text{Divergent, } \infty}$

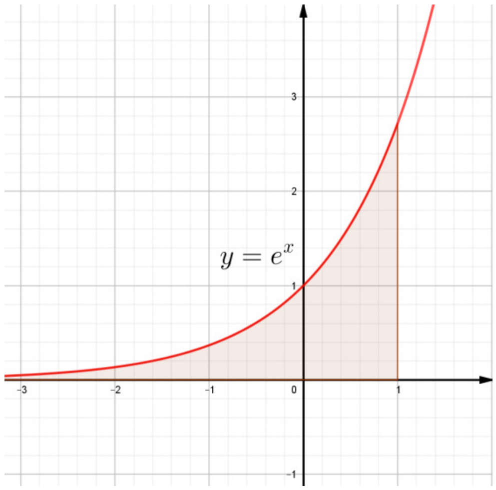
16) $\int_{1/4}^{1} \frac{1}{4y-1}\, dy$ $\boxed{\text{Divergent, } \infty}$

17) $\int_{0}^{\pi} \sec x \, dx$ $\boxed{\text{Divergent, } \infty}$

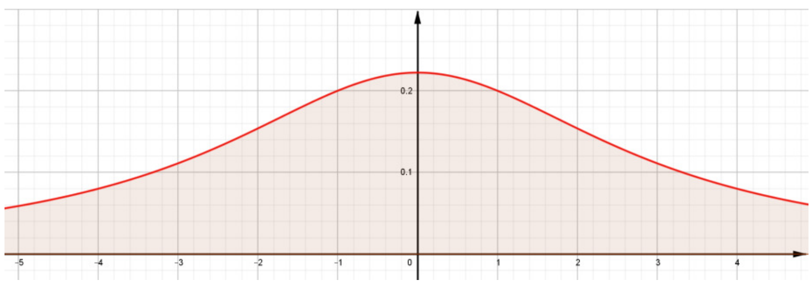
18) $\int_{0}^{4} \frac{dx}{x^2+x-6}$ $\boxed{\text{Divergent, } -\infty}$

Sketch the region and find its area (if the area is finite).

19) $S = \{(x, y) \mid x \leq 1,\ 0 \leq y \leq e^x\}$ $\boxed{e}$



20) $S = \left\{(x, y) \mid 0 \leq y \leq \dfrac{2}{x^2+9}\right\}$ $\boxed{\dfrac{2\pi}{3}}$



21) A manufacturer of light bulbs wants to produce bulbs that last about 700 hours but, of course, some bulbs burn out faster than others. Let $F(t)$ be the fraction of the company's bulbs that burn out before t hours, so $F(t)$ always lies between 0 and 1.

a) What is the meaning of the derivative $r(t) = F'(t)$?

b) What is the value of $\int_0^\infty r(t)\, dt$? Why?

> $a)$ $r(t) = F'(t)$ is the rate at which the fraction $F(t)$ of burnt-out bulbs increases as t increases. This could be interpreted as a fractional burnout rate.
>
> $b)$ $\int_0^\infty r(t)\, dt = \lim_{x\to\infty} F(x) = 1$, since all of the bulbs will eventually burn out.